Benefits of Wiener Beamforming for Ultrasound Imaging

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***Abstract*—CF- and MV-based methods (and combinations thereof) have been suggested for improved ultrasound imaging. Recently, it was shown that the CF can be interpreted as a Wiener postfilter and the CF-MV combination as a Wiener beamforming. However, no real study of the advantages of using Wiener beamforming has been presented. In this paper we give an in-depth study of the behavior of the Wiener methods, and their expected benefits for ultrasound imaging. We will cover the advantages and disadvantages of the different Wiener beamformers, as well as their robustness.**

***Index Terms*—Ultrasound imaging, adaptive beamforming, coherence factor, Wiener beamformer**

I. INTRODUCTION

Adaptive beamformers are data-dependent algorithms that can be used to enhance the quality of ultrasound images beyond what can be achieved with conventional delay-and-sum (DAS) beamforming. The increased performance is obtained by adjusting the beamformer behavior to the situation at hand, optimally exploiting all available degrees of freedom. The two classes of adaptive beamformers that have received the most attention in ultrasound imaging are the Minimum Variance (MV)- and the Coherence Factor (CF)-based algorithms. SOMETHING ABOUT MV BEAMFORMERS WITH REFERENCES. SOMETHING ABOUT CF BEAMFORMERS WITH REFERENCES. The MV and CF techniques were theoretically united in [**?**], as the components of the Minimum Mean Square Error (MMSE) Wiener beamformer.

Although a lot of research has been done on the MV and CF methods, there are still several unanswered questions about their performance and the performance of Wiener beamformers. In this paper we attempt to give an exhaustive summary of the various performance enhancements that are available in the Wiener beamformer framework.

- Collect all references on CF and Wiener methods...
- Does the Wiener beamformer increase resolution?
- Does the Wiener beamformer decrease sidelobes?
- What is the danger of "evaluation by visual inspection"? (Artificially low sidelobes etc.)
- Does the Wiener beamformer decrease RMSE? Compare to DAS and MV for point-in-speckle simulations. Compare to DAS without speckle.
- Does the Wiener beamformer decrease grating lobes? (Cmp. PCF) (Is this a separate paper to be cited?)
- Does the Wiener beamformer increase SNR? **Point:** Pointwise vs. regional SNR improvement.
- Can the Wiener beamformer extract information (points) buried in spatially white noise?
- What is the difference between assuming noise to be spatially white or not? (i.e. $\hat{\mathbf{R}}_p = \hat{\sigma}^2\mathbf{I}$ vs. $\hat{\mathbf{R}}_p = \hat{\mathbf{R}} - \vec{m}_x\vec{m}_x^H$
- *Very specific formulas* for calculating postfilters, *no ambiguities*.
- Is a problem with all CFs that they are based on irrelevant separations of signal and noise? Irrelevant because we cannot identify anything else than what we have already removed as noise... Is the hidden hypothesis that "a lot of noise remove indicates a lot of noise left"? (Apparently... but is it necessarily a bad hypothesis? May be OK for spatially white noise, because there is a fixed relationship between initial and residual noise.)

II. BACKGROUND

A. Array and Signal Model

$$\vec{x}[n] = \vec{s}[n] + \vec{p}[n] \in \mathbb{C}^{M,1} \tag{1}$$

B. The Minimum Variance Distortionless Response Beamformer

The Minimum Variance Distortionless Response (MVDR) beamformer...

$$\vec{w}_{MV} = \text{argmin}_{\vec{w}} E\left\{\left|\vec{w}^H\vec{x}\right|^2\right\} \text{ subject to } \vec{w}^H\vec{x} = 1 \tag{2}$$

$$\vec{w}_{MV} = \frac{\mathbf{R}^{-1}\vec{d}}{\vec{d}^H\mathbf{R}^{-1}\vec{d}} \tag{3}$$

C. The Wiener Postfilter

The Wiener postfilter H_{MMSE} associated with the weight vector $\vec{w}$ is the scalar that minimizes the Mean Squared Error (MSE) of the beamformer output $y = \vec{w}^H\vec{x}$ when multiplied with it:

$$H_{MMSE}(\vec{w}) = \text{argmin}_H E\left\{\left|A - H\vec{w}^H\vec{x}\right|^2\right\} \tag{4}$$

IMPLEMENTATION ASSUMING SPATIALLY WHITE NOISE

$$\hat{H}_{MMSE}(\vec{w}) = ... \tag{5}$$

GENERAL IMPLEMENTATION WITHOUT ASSUMING SPATIALLY WHITE NOISE

$$\hat{H}_{MMSE}(\vec{w}) = ... \tag{6}$$

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D. The Wiener Beamformer

The Wiener beamformer is the weight vector $\vec{w}_{MMSE}$ that minimizes the MSE of the beamformer output:

$$\vec{w}_{MMSE} = \text{argmin}_{\vec{w}} E\left\{\left|A - \vec{w}^H\vec{x}\right|^2\right\} \tag{7}$$

Relationship to MVDR:

$$\vec{w}_{MMSE} = H_{MMSE}(\vec{w}_{MV})\vec{w}_{MV} \tag{8}$$

IMPLEMENTATION

$$\vec{w}_{MMSE} = ... \tag{9}$$

III. PERFORMANCE OF WIENER BEAMFORMING

- Cysts (massive and anechoic) in speckle; like amplitude estimation paper. Compare to DAS and MV.
- Point scatterers - probability of resolution.
- Low-magnitude scatterers in sidelobes of original point scatterers.

A. Phase Aberrations

- Two cases; cyst (CR/CNR) and point scatterer (HPBW/resolution, SLL).
- Different from other metrics; model deviation.
- Phase errors modeled as increased white noise, ref. Van Trees.
- Compare to GCF? (Done in separate paper, comment only?
- Compare to MV?
- Point: Unlike CF, WBF is not designed to counteract phase aberrations. If/as p.a. are detected as decreased SNR, then it will have an effect.
- Find CR as a function of aberration profile strength.
- Is the increased performance in the case of phase aberrations (approximately) identical (barring a scaling factor) to that in the case of no phase aberrations? If so; does this mean that CF/Wiener is not a phase aberration reduction method?

B. Sidelobe Reduction/Grating Lobe Reduction

1) Refer to claims for PCF wrt. grating lobe reduction. Study in a different letter?
2) Refer to all claims on sidelobe reduction. Present point scatterer with low sidelobes, and point scatterer with other, low-magnitude scatterer positioned in the sidelobes. Compare to DAS.

C. Point Magnitude and Resolution

- Demonstrate that CF does not reduce smallest angle of separation, but increases degree of separation (peak-to-dip ratio) beyond this point.
- Can possibly increase POR for very low SNR?
- Point scatterer RMSE in white noise or speckle vs. DAS and MV.

D. Speckle Magnitude and Contrast

- Cyst contrast.
- Contrast-to-noise ratio.
- Absolute speckle level RMSE (USSYMP 2009 Roma).
- Transition length at interface.
- Speckle mu/sigma?

E. Signal-to-Noise Ratio

WiP does not affect pointwise SNR; signal and noise are equally attenuated:

$$SNR_{pt,WiP} = \frac{|y_s|^2}{|y_p|^2} = \frac{|H|^2\left|\vec{w}^H\vec{s}\right|^2}{|H|^2\left|\vec{w}^H\vec{p}\right|^2} = SNR_{pt,DAS} \tag{10}$$

WiP can affect the *average-signal-to-average-noise-power ratio*, i.e. the relative amounts of signal and noise power inside some region:

$$SNR_{reg,WiP} = \frac{\sum_{n\in N_{reg}}|y_s[n]|^2}{\sum_{n\in N_{reg}}|y_p[n]|^2} = \frac{\sum_{n\in N_{reg}}|H[n]|^2\left|\vec{w}^H\vec{s}[n]\right|^2}{\sum_{n\in N_{reg}}|H[n]|^2\left|\vec{w}^H\vec{p}[n]\right|^2} \tag{11}$$

Although it is intuitively attractive, this is not necessarily a good metric. Note especially that the optimal solution is given by

$$H_{opt}[n] = \delta[n-n_0] \text{ where } n_0 = \text{argmax}_{n\in N_{reg}}\frac{|y_s[n]|^2}{|y_p[n]|^2} \tag{12}$$

IV. DISCUSSION

- Should we "always" use the Wiener postfilter (beamformer) instead of DAS (or MV)? **There's really no reason why not:** Low additional complexity and decomposable output.
- How is the improvement of WBF over MV compared to that of WPF over DAS?
- How about "mixing" MV- and DAS-based CFs. Are there any arguments for not using all DAS or all MV?
- What improvements are "actual" and what are "visual tricks"? Adaptive "sidelobes"? Comment on what can be extrapolated from simple cases for nonlinear adaptive techniques. Make argument to compare with MV, as the DR requirement makes the results valid.
- What is resolution? Does Wiener algorithm improve it? Initial separation vs. degree of separation. Proof of no resolution enhancement?
- How do we measure the performance of Wiener-type methods? Adaptive beamformers in general? Representative scenarios, "superposition"-type arguments and such?
- If contrast is the main argument behind Wiener methods, is the improvement "global" or "local"? Can we use one global adaptive contrast transformation?
- What about the white noise assumption? Frobenius-norm distance from white noise matrix any indicator?
- In general; how does the adaptive performance of Wiener methods differ from MV conceptually (subtraction vs. scaling) and practically (with respect to metrics like CR etc.). Maybe MV yields (point scatterer) resolution, WBF

  yields contrast? What about interfaces? (Massive and anechoic cyst edges etc.)
- Computational complexity vs. MV.

V. CONCLUSION

REFERENCES

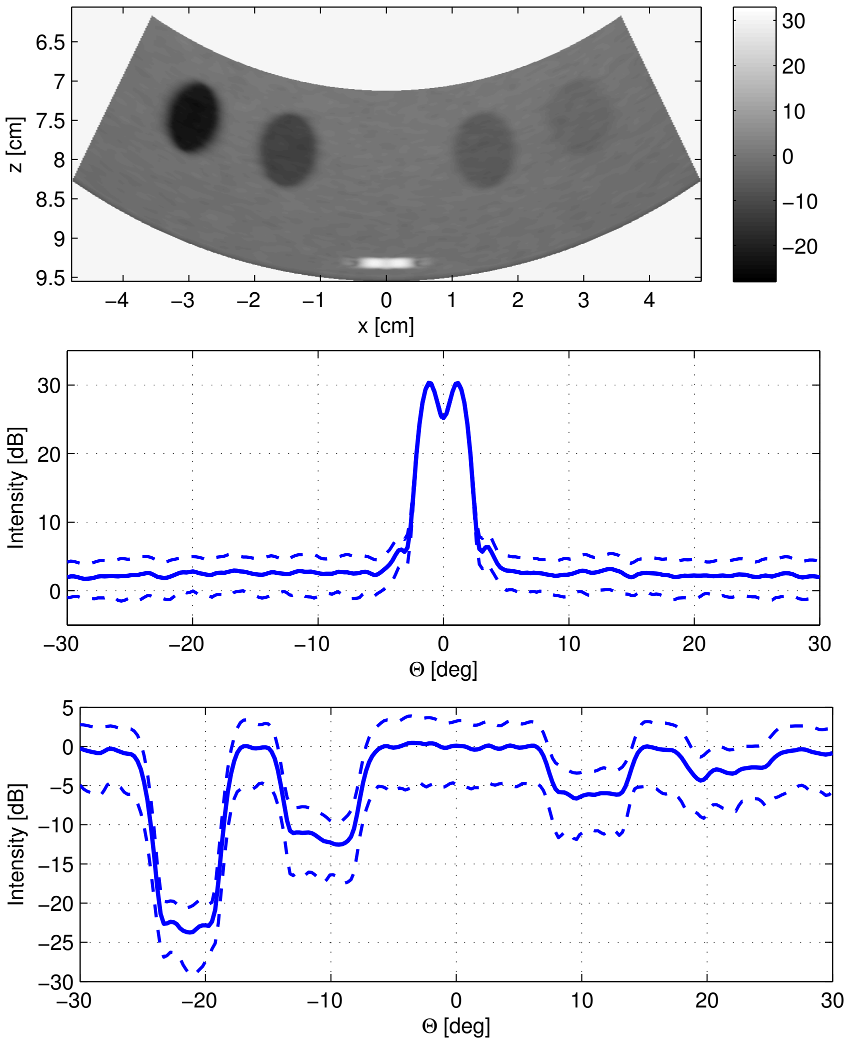

Fig. 1. DAS, negative amplitudes

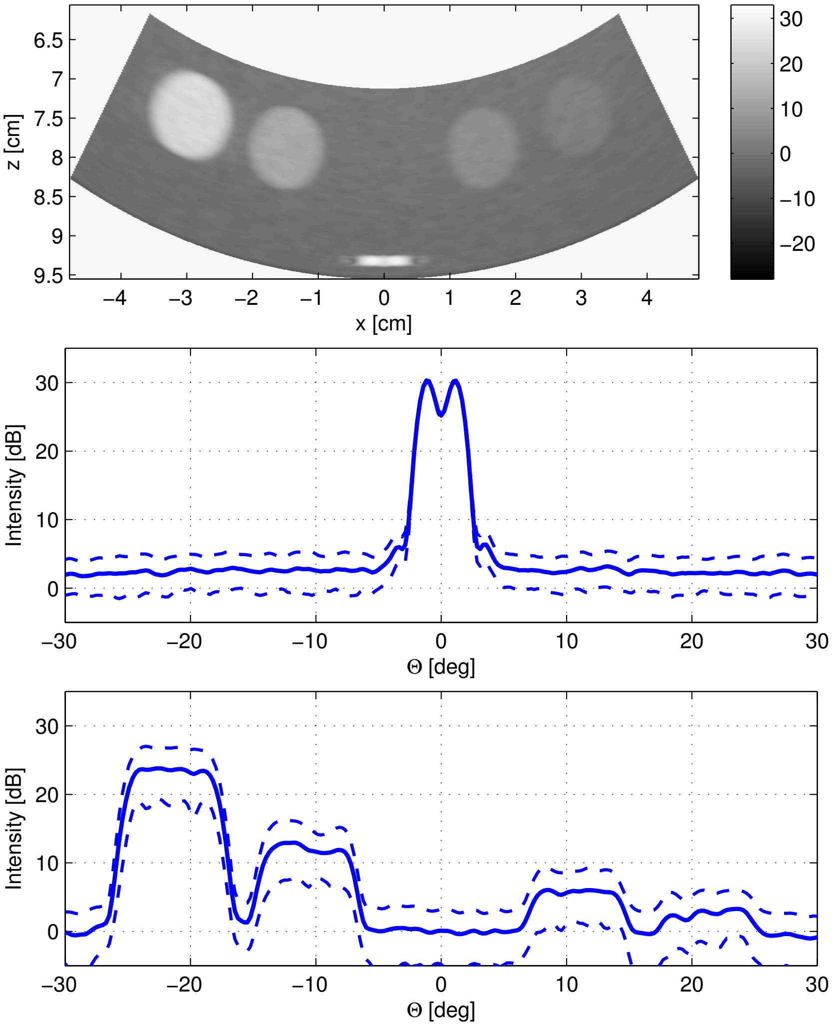

Fig. 2. DAS, positive amplitudes

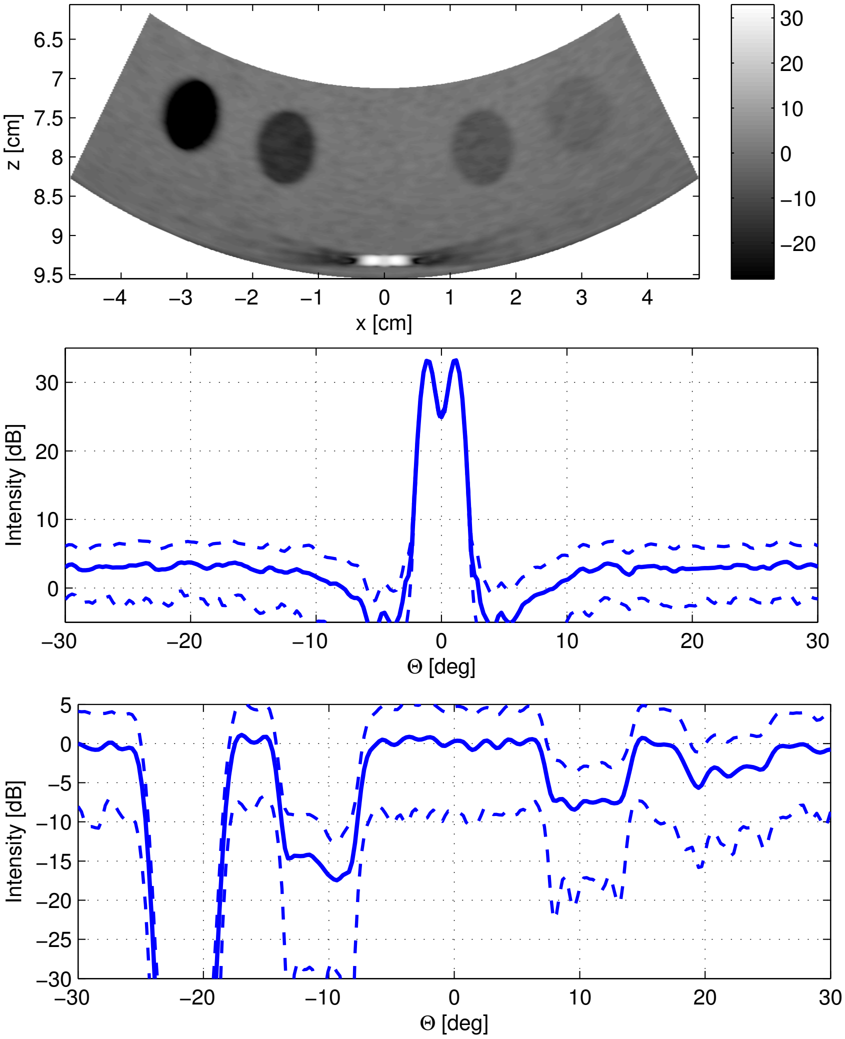

Fig. 3. CF, negative amplitudes

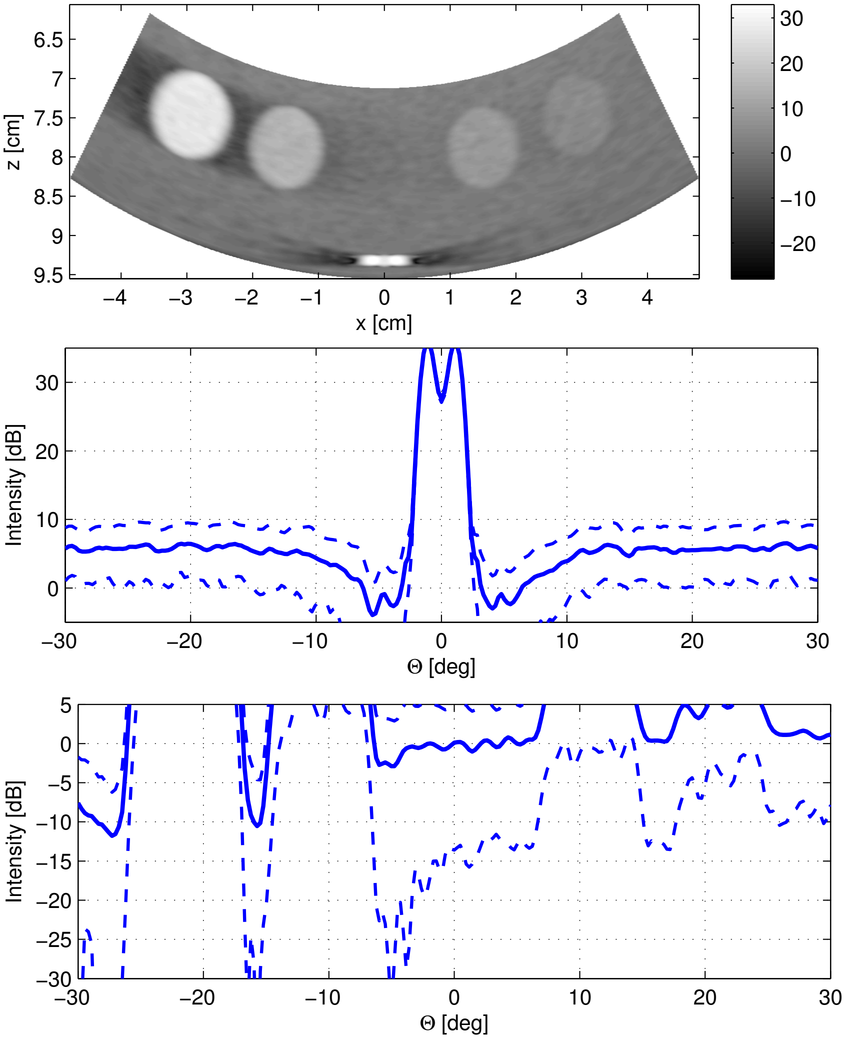

Fig. 4. CF, positive amplitudes

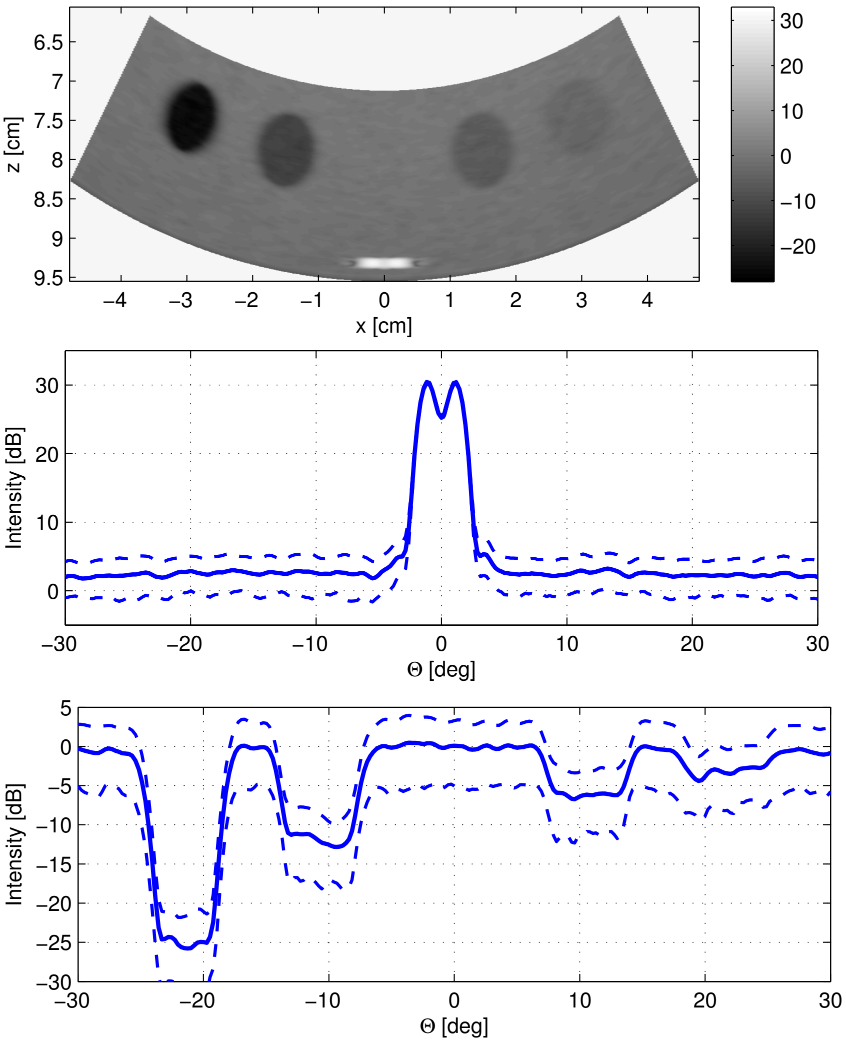

Fig. 5. WiP, negative amplitudes

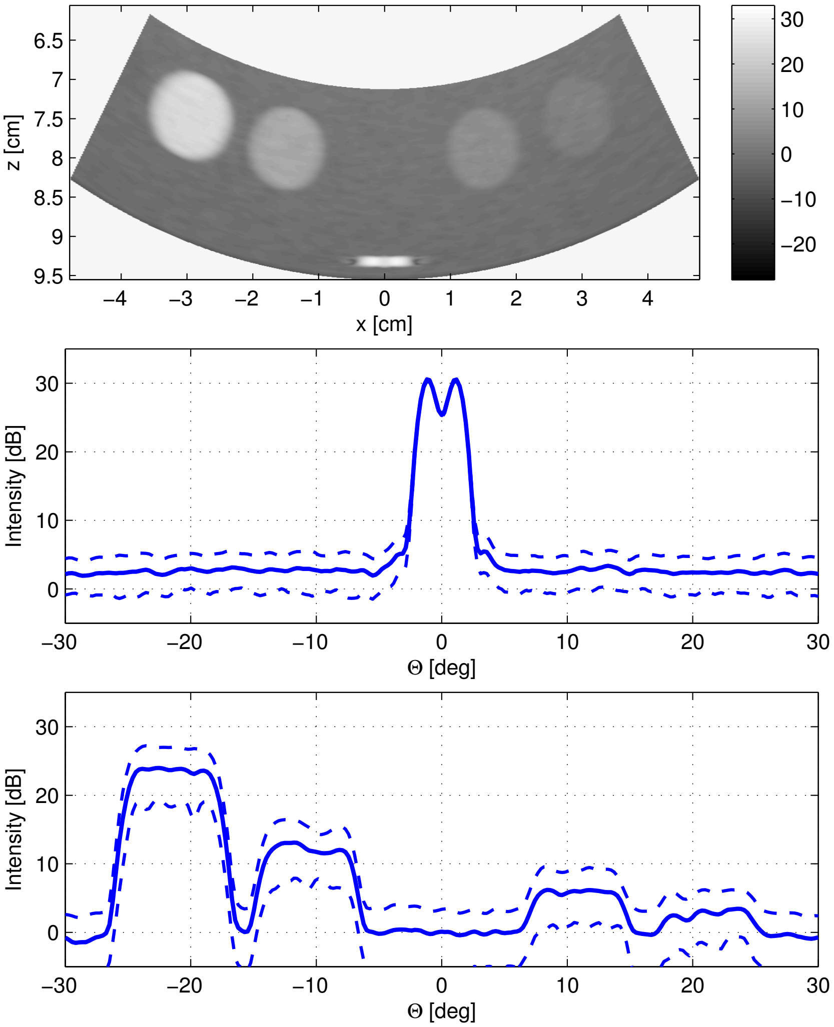

Fig. 6. WiP, positive amplitudes